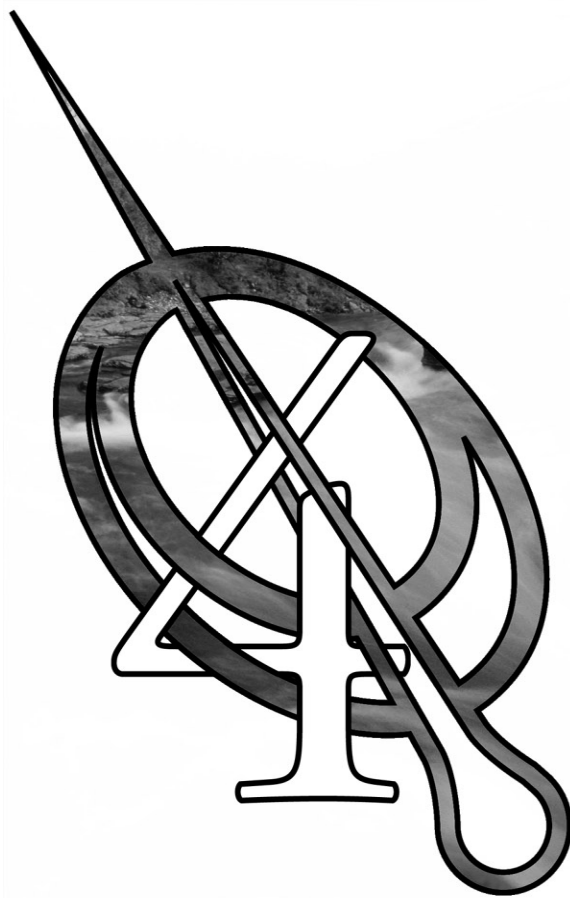


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# Salvaging Descartes' *Géométrie*

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After investigating various mathematical, historical and philosophical sources concerning René Descartes' *La Géométrie*, I am convinced that this work is not understood in accordance with the original intent of its author. By using Descartes' philosophical works, I'll offer a different interpretation of this publication, and explain how poor interpretations have resulted in paradoxically beneficial impacts on mathematical knowledge. My aim is to connect early 17<sup>th</sup> Century Western intellectual discourse with the intent of Descartes in publishing this mathematical work, while looking beyond any mere categorical relation between Descartes and the church, or Descartes and the skeptics. Specifically, my goal is not to denounce these relations, but to broadly consider the interpersonal social and scientific discourse of Descartes' day, along with his personal experiences and philosophy, while attempting to construct a more accurate interpretation of *La Géométrie*. Such an interpretation is made possible by utilizing a method that both mathematicians and philosophers can easily follow, and this merely requires being familiar with certain key aspects of Descartes' *Discourse on the Method of Reasoning Well and Seeking Truth in the Sciences*.

René Descartes (1596-1650) is frequently considered the most influential mathematician and philosopher of the early 17th Century. To modern scholars he characterizes the impetus behind what is now seen as a dramatic shift from medieval traditions towards scientifically oriented thought. Nevertheless, Descartes was not alone in his academic and intellectual activities—mathematics and philosophy flourished in early 17<sup>th</sup> Century Europe. To get an idea of the extent of achievements during this period, consider a list of relatively well-known French mathematicians, many of whom were in regular contact with Descartes or had a special influence on his studies: François Viète, Pierre Hérigone, Claude Bachet, Jean-Baptiste Morin, Pierre Vernier, Etienne Pascal, Marin Mersenne, Girard Desargues, Pierre Gassendi, Jean Beaugrand, Albert Girard, Claude Hardy, Pierre de Carcavi, Pierre de Fermat, Florimond de Beaune, Jacques de Billy, Gilles Roberval, Ismael Boulliau,

F r e -

nicle de Bessy, Honoré

Fabri, Jacques Le Tenneur, Antoine Arnauld, Gabriel Mouton, Jean Picard, Claude Dechales, Blaise Pascal, Jean Richer, Nicolas Malebranche, Philippe de la Hire, Bernard Lamy, and Jacques Ozanam. In addition, many great minds in Germany (Kepler), Italy (Galileo, Cavalieri), and Britain (Brigs, Harriot, Oughtred, Napier) were also making advancements in mathematics at this time. *All of these people had significant impacts on mathematics.*

Descartes published the *Discourse* in 1637. Originally it was published in French. This fact alone is curious, insofar as learned writings of the time were typically in Latin. Even more intriguing is the fact that Descartes' name as author was absent from the work and its appendages in the first edition, and he received nothing for the publication except 200 free copies to distribute amongst his colleagues.<sup>1</sup> Several interpretations of these facts have been made. Many scholars suggest that the *Discourse* was not intended to be the formal philosophical treatise that it has become. In it, Descartes autobiographically communicates the discovery of his method of systematic doubt—a method used to reach valid conclusions in scientific inquiry. A more thorough investigation into this matter, however, shows that the motivations behind the publication of the *Discourse* were slightly more complicated.

It is all too common today to equate the Renaissance with a battle between the Catholic Church and skepticism or reasoning. In fact, some modern texts even argue that several key figures in the history of scientific reasoning made names for themselves on the basis of their opposition to the Catholic Church, and sometimes nothing more.<sup>2</sup> This is unquestionably an oversimplification of the actual practices by both academic and religious institutions of the Renaissance.

We know from Descartes' correspondence that he considered several issues to be important (not just the relationship of the Catholic Church with science) when he wrote the *Discourse*. While it is known that Descartes was a self-proclaimed catholic (although he lived with a protestant mistress in protestant Holland when the *Discourse* was published), many modern scholars are lead by this in attempts to rectify the position of the Catholic Church with what they see as anti-Catholic views in Descartes' writings. This sometimes leads to the assertion that Descartes professed to be catholic so that his works would be accepted  
and not

banned by the church. (Actually, his works were put on the index after his death.) However, it should be noted that Descartes had more problematic encounters with the Protestant theologians of Holland than he did with the Catholic Church. At one point, booksellers in Holland were forbidden from printing or selling any of Descartes' works until he appeared before the court in Utrecht to defend the charge that he was an atheist.<sup>3</sup> Such facts lead us to question what should and what should not be considered during attempts to interpret works such as the *Discourse*. Do these facts about religion shape the social context in which the *Discourse* was written, and if so, do they help us gain a better understanding of what Descartes hoped to achieve by writing his treatise on method? Of course they do. But how?

Theodor W. Adorno and Michel Foucault offer a possible solution to this dilemma. One might ask: "What dilemma?" The answer: the dilemma lies in our assumptions regarding the nature of continuity in historical disciplines. According to Foucault, the notions of tradition, influence, development, and evolution "diversify the theme of continuity."<sup>4</sup> These notions cannot be trusted—they have no privileged access to explaining their own foundations. For example, a broad claim about the nature of the Catholic Church's intellectual suppression tactics in Descartes' time assumes notions of tradition and influence, which might in turn be used as the basis for explaining why Galileo, or anyone for that matter, encountered intellectual roadblocks. In other words, rupture and continuity are not themes to be found in history through extensive research, as is commonly assumed by many historians of science and ideas. Rupture and continuity are typically assumed, and the research follows by attempting to justify these assumptions. That is, historians commonly assume that history is shaped by ruptures *or* continuity, and their investigative efforts frequently aim to justify such claims. Thus, in Descartes' case, we may use the historical traditions or notions of religious and scientific debates in an attempt to explain why these debates even occurred; but this, as Foucault points out, is circular reasoning. Another excellent example of this idea can be seen in the way Descartes is perceived as "emancipated from tradition" (we made this assumption earlier when

conside-  
ring Descartes as somehow  
individually definitive of the division between  
pre-modern and modern philosophy). Adorno deals ex-  
plicitly with this issue as it relates to Descartes: “It is questionable  
whether the idea underlying this position—that tradition, what is not  
known at first hand, should be spurned in face of the immediacy of lived  
experience—whether this motif, which we take almost for granted, is really  
so valid.”<sup>5</sup> One of the perplexing aspects of Descartes’ works as they are inter-  
preted today is that we can simultaneously identify him with revolutionary and  
antiquated traditions. While we see Descartes’ method of reasoning as ground-  
breaking, he considered the examples he used to promote his method (e.g., his  
mathematics) to be rooted in the intellectual achievements of antiquity.<sup>6</sup>

The central theme here is that the entire realm of discourse in Descartes’ day  
provides us with a better perspective to understand his works than our knowledge  
of Enlightenment traditions, (anti-)religious influences, intellectual developments,  
or conceptual (r)evolutions. It is not my claim that these notions of continuity  
are obviously incorrect. However, in the spirit of the *Discourse* we must doubt their  
validity and applicability in order to hypothetically and scientifically consider them  
fallacious. In this simple manner the method presented in the *Discourse* should be  
used to study the *Discourse* itself. After all, the method of systematic doubt was not  
offered by Descartes solely as a means to ‘reason about reason’—it essentially was  
and is a practical guide to problem solving.

As Foucault points out, “We must renounce all those themes whose function is  
to ensure the infinite continuity of discourse and its secret presence to itself in the  
interplay of a constantly recurring absence.”<sup>7</sup> In other words, while the notions of  
beginning and rupture are likely to distance us from the most historically enlighte-  
ning portrayal of Descartes’ philosophy, so is the opposite notion of the ‘not-said.’  
This makes the task of the historian much greater and, upon closer inspection,  
renders the possibility of a complete and ultimately correct interpretation incredi-  
bly improbable. Nevertheless, Foucault’s method of categorizing history based on

discourse alone (known as his “archaeology”) will often show why the commonly accepted interpretation of an event is erroneous.

This is demonstrated by Descartes' famous encounter with Chandoux in 1628. Chandoux, along with many others, claimed that science could only be based on mathematical probabilities. (It is interesting to note that there were incredible achievements made in probability theory during this time.) Anyone who has read Descartes' *Discourse* should immediately recognize that this view is not consistent with Descartes' position. But why? At this point, most texts will state that while meeting Chandoux in Paris, Descartes claimed certainty was the basis of scientific investigation. This historical claim presents no problems as it is clearly grounded in historical discourse, taking the form of a conversation between Descartes and Chandoux. The problem arises when the further claim is presented that Descartes “had a method for obtaining such certainty,” because then the distinction between Chandoux and Descartes becomes vague. Did they not both seek certainty, and, furthermore, wasn't this the basis of scientific investigation for both of them? While Descartes had such a method, we must emphasize that the method was primarily a function of certainty, and certainty was secondly a result of the method. Through this interpretation we must praise Descartes' method before we praise the results it obtains.

The *Discourse on Method* was originally 78 pages, comprising roughly one-sixth of the original publication. Attached to the *Discourse* were three appendages: *La Dioptrique*, *Les meteors*, and *La Géométrie*. These were included as practical examples of the method of scientific reasoning outlined in the *Discourse*. Although discovered earlier by Snell, *La Dioptrique* contained the first known publication of the law of refraction, in addition to information about the anatomy of the human eye and the shape of telescope lenses. *Les meteors* contained scientific explanations of rainbows, snow crystals, the size of raindrops, and the relation between thunder and lightning. *La Géométrie*, at least *prima facie*, seemed more old-fashioned insofar as it appeared to augment existing mathematical traditions.<sup>8</sup> It is important to note that a large part of *La Géométrie* deals specifically with a problem that

worried  
 mathematicians for a  
 long time before Descartes—the  
 Pappus problem.<sup>9</sup> In *La Géométrie* Descar-  
 tes presented a new method of solving the pro-  
 blem. Nevertheless, readers had difficulty seeing how  
 the three appendages were related to the *Discourse* and,  
 consequently, they were removed in later printings. Ever  
 since, *La Géométrie* has rarely been published and attempts to  
 connect it to the *Discourse* have dwindled.

*La Géométrie* alone constituted 116 pages of the original pu-  
 blication of the *Discourse*. It is composed of three separate sec-  
 tions: “Problems the construction of which requires only straight  
 lines and circles,” “On the nature of curved lines,” and “On the  
 construction of solid and supersolid problems.” A description  
 of each section (which I will now provide) is required for a  
 summary view of the whole. For the sake of clarity I will  
 refrain from using any mathematical terms which,  
 although superficially impressive, could easily be  
 confusing.

The first section, “Problems  
 the construction of

w h i c h  
 requires  
 onlystraight  
 lines and  
 circles,” opens  
 boldly: “Any pro-  
 blem in geometry can  
 easily be reduced to such  
 terms that a knowledge of  
 the lengths of certain lines is suf-  
 ficient for its construction.” Descartes  
 proceeds to show how the common arithmetic  
 functions of addition, subtraction, multiplication, division,  
 and extraction of root can be carried out purely geometrically with a straight edge  
 and compass and composed completely in a plane diagram. The generality of this  
 view introduced an idea that would have a central theme in Descartes’ geometry.  
 Instead of interpreting the multiplication of two lines as an area, as had been done

since ancient times, Descartes' new geometrical method showed that the multiplication of two lines could instead be abstracted and simply create a new line. By multiplying in a third line, the ancients would have postulated a three-dimensional volume. To Descartes, it was again simply another line. At the point of multiplying in a fourth line, the ancient interpretation ceases to follow intuition, while Descartes' interpretation does not change.

This is of significant importance when Descartes gives his solution to the Pappus problem. Contrary to what many texts claim, the Pappus problem had been solved with a general solution before the time of Descartes. However, this solution was largely ignored as it, according to Pappus himself, "signified nothing comprehensible" in the sense that it allowed multiplication of lines beyond the known dimensional space of volumes. While there is evidence that ancient mathematicians such as Diophantus had used higher orders of multiplication, they were always limited in their capacity to grasp the theoretical result. In efforts to explain their results in comprehensible terms, they were referred to as "squared-squared" or "cubed-cubed" and so on. This is one reason why Descartes' solution was revolutionary. It allowed for a general solution to the problem that was based more in abstraction than intuition.

The second section, "On the nature of curved lines," is the least complete and most interpretively demanding portion in *La Géométrie*. At the same time, it is incredibly complex and yields many of Descartes' greatest known contributions to mathematics. This section is also concerned with the Pappus problem, but in a theoretical sense in which Descartes merely questions the conditions under which this and other problems are capable of being solved. In order to answer this, he considers and critiques the conditions under which Pappus judged the problem capable of a comprehensible solution. It is clear that Pappus' classification of the lines for which the problem was capable of solution is based principally on dimensionality, and secondly on line structure.

As Descartes has shown, he is now capable of avoiding the complicated issue of dimensionality altogether. Thus, all of Descartes' contemplation is placed on line structure. He continues and formulates a classification of acceptable lines and curves for not only the Pappus problem, but for geometry in general. This specific aspect of *La Géométrie* is not entirely remarkable, for such classifications had been attempted previously. In fact, Descartes' classification was later considered inconsistent and therefore unacceptable. However, the reasons why he actually grouped specific curves and lines as acceptable or not have largely been forgotten or ignored.

Descartes grouped curves and lines into two groups. First, the "geometrical" curves and lines are those which are known with absolute certainty. "Mechanical" curves are those which are not known with absolute certainty. According to Descartes, even extremely complex curves, if known with certainty, should be allowed in geometry. By "known with certainty," he is referring to the path of the curve or line, not its position relative to coordinates. In fact, Descartes never used coordinates to describe the path of curves or lines. Although he is generally given credit for the first practical use of coordinates and abscissa, these appear nowhere in his writings. They are arguably better attributed to Leibniz and Newton, although they could be viewed as the result of Descartes' geometrical methods applied by other mathematicians. (It is interesting to note that abscissa in the time just after Descartes were not required to be perpendicular.) In any case, Descartes never used them. He did, however, make extensive use of a line as a reference point, which greatly aided his geometrical constructions. We should keep in mind that, while this appears to be close to the formal idea of a coordinate system with abscissa, Descartes simply never made the latter notion explicit in his work. This also helps explain the problem that Descartes faced with negative and complex numbers. I will discuss this further when I discuss section three of the *La Géométrie*.

According to Descartes, geometrical curves all share a special resemblance to a straight line. By this, he claimed that all points on such geometrical curves could be expressed in definite relation by means of a single equation. On the other hand, curves which admit two simultaneous unrelated motions, according to Descartes, are by their very nature vague and therefore "mechanical," since no connection

between these motions can be determined.

It is important to understand that curves admitting two motions, which are capable of being reconciled into one motion, would be geometrical. Descartes' classification is therefore rather simple and precise, or, to use an appropriate phrase, "clear and distinct." I believe this notion plays a key role in *La Géométrie*, one which is not usually considered.

The other major contribution from Descartes in section two is a general method for finding the normal (i.e., the perpendicular) of a curve at a given point. After finding the normal, it is possible to calculate the slope of the tangent line to the curve at that point. This is the chief aim of derivative calculus.

Although Descartes' method used completely algebraic means, it is notable for its generality and uniqueness.

While this may seem like a minor contribution, it must be understood that some of the best mathematicians of all time had worked on this problem and were incapable of providing such a general solution. Descartes himself said, "I dare say that this is not only the most useful and the most general problem in geometry that I know, but even that I have ever desired to know."<sup>10</sup>

In the third section, "On the construction of solid and supersolid problems," Descartes focuses more on algebra than on geometrical problems. The center of attention of this section is primarily devoted to the methods of simplification. Descartes shows how his methods can be used in a generalized approach to solve problems that were commonly restricted by dimensionality. He emphasizes that using the simplest curve possible to solve a problem will yield the best result. He also is the first to explicitly state what has come to be known as "Descartes' rule of signs." This is simply an upper bound to the number of possible correct solutions to an equation, based on specific qualities of the equation itself. Descartes' rule of signs considers both positive and negative numbers (sometimes referred to by Descartes as "real" and "false" numbers) as possible solutions to the equation, but is incapable of stating what these solutions are. Several mathematicians, notably Cardan and Harriot, had made use of this rule before Descartes, but never stated it formally as it is presented in *La Géométrie*.

A f t e r  
d e t e r m i n i n g  
t h e t o t a l n u m b e r o f  
s o l u t i o n s t o a n e q u a t i o n,  
D e s c a r t e s s e e k s a r e f i n e m e n t o f  
t h e r u l e o f s i g n s w h i c h w o u l d a l l o w  
h i m t o o n l y s t a t e t h e n u m b e r o f “r e a l,”  
p o s i t i v e s o l u t i o n s. I t i s a t t h i s p o i n t w h e r e  
D e s c a r t e s m a k e s a m i s t a k e t h a t w o u l d l e a d t o  
m u c h o f t h e c o n f u s i o n t h a t s u r r o u n d s *La Géométrie*,  
f o r h i s r e f i n e m e n t t u r n s o u t t o b e i n c o r r e c t. A s J o h n  
W a l l i s p o i n t e d o u t i n 1685, “T h i s r u l e i s e i t h e r a m i s t a k e,  
o r a n i n a d v e r t e n c e, f o r i t m u s t b e t a k e n w i t h t h i s c a u t i o n,  
t h a t i s, t h a t t h e r o o t s a r e r e a l, n o t [c o m p l e x].”<sup>11</sup> T h i s o v e r s i g h t  
b y D e s c a r t e s h a s l e d s o m e h i s t o r y o f m a t h e m a t i c s t e x t s t o c l a i m  
t h a t D e s c a r t e s c o u l d n o t w o r k w i t h n e g a t i v e o r c o m p l e x n u m b e r s.  
T h i s a s s u m p t i o n t u r n s o u t t o b e f a l l a c i o u s. T h e t r u t h o f t h e m a t t e r  
i s, w h i l e D e s c a r t e s s o u g h t t o w o r k o n l y w i t h “r e a l” n u m b e r s, p o s i t i v e  
i n t e g e r s, h e a d m i t t e d n e g a t i v e a n d c o m p l e x n u m b e r s i n a l g e b r a i n t h e  
s a m e s e n s e t h a t h e a d m i t t e d m e c h a n i c a l c u r v e s i n g e o m e t r y. N e g a t i v e  
a n d c o m p l e x n u m b e r s a r e v a g u e, n o t c e r t a i n n o t i o n s. (T h e y a r e s t i l l v a g u e  
n o t i o n s t o u s t o d a y.) J u s t a s D e s c a r t e s d o u b t e d t h e c e r t a i n t y o f m e c h a n i c a l  
c u r v e s, h e d o u b t e d t h e c e r t a i n t y o f a l l b u t “r e a l” n u m b e r s, a n d  
t h e r e f o r e w o u l d n o t u s e t h e m.

It is hard to determine what Descartes hoped to accomplish with the final section of *La Géométrie*. Although it shares some interesting results, its structure resembles a rough draft more than a finished product. Generally, this is the view that most mathematicians take in respect to *La Géométrie* in its entirety. It is horribly arranged, and it is hard to know at any point in the text what Descartes actually seeks to accomplish. This is only made worse by Descartes’ exclusion of proofs with the hope of making their discoveries more enjoyable for the reader. On this ground, it is completely understandable why the work was subsequently excluded from later publications of the *Discourse*. Nevertheless, I believe that it is precisely the separation of the *Discourse* and *La Géométrie* that makes the latter generally impossible to interpret authentically today.

I stated earlier that *La Géométrie* could be understood without the usage of confusing mathematical terms and notation. This is because Descartes’ original intention was to use it as an example of the method provided in the *Discourse*. While I agree that *La Géométrie* is horribly arranged, in

the sense that readers find no explicit relation to the *Discourse*, I find the evidence more than compelling that later additions to *La Géométrie*—mostly attempts by others to explain the mathematical results found by Descartes—did more injustice to the work simply because they ignored the reasons for which it was first published.

Sections two and three of *La Géométrie* give us the best understanding of how Descartes used his method of certainty in geometry. Recall that during his classification of geometrical and mechanical curves, Descartes claimed that curves composed of two motions incapable of being reconciled with each other must be mechanical in nature. Specifically, Descartes argued that curves composed of straight *and* curved segments must be mechanical, as straight and curved motions were impossible to unite in one certain motion. It has been suggested that Descartes' motivation for making this distinction was the inability to reconcile the most basic of all curves, the circle, with its own radius. The search to find a rational value for  $\pi$ , or prove there was none, had long been an important part in mathematics before Descartes. (In fact, to this day there are those who attempt to prove that  $\pi$  is rational.)

Descartes does mention the very well known problem of “circle squaring” explicitly in a letter to Mersenne just one year after the first publication of the *Discourse on Method*. According to Descartes, “one cannot admit [in geometry] lines which are like strings, that is, which are sometimes straight and sometimes curved, because the proportion between straight lines and curved lines is not known and I also believe it cannot be known by men, so we cannot conclude anything exact and certain from it.”<sup>12</sup> Unfortunately, he does not mention this explicitly anywhere in *La Géométrie*. Nevertheless, if one assumes that Descartes' classification of mechanical curves was based, at least in part, on the problem of squaring the circle, an interpretation of *La Géométrie* which gives priority to the method of certainty found in the *Discourse* (one that explains mathematical results as products of the method) is ultimately possible. This approach would also help to explain why *La Géométrie* is so unorganized, yet included originally in a work with glorified certainty and order.

A likely objection would be that squaring of the circle was not rigorously proved impossible until 1882 (by Lindemann), and Descartes was incapable of knowing in particular whether it was possible or not. This is true. However, under the interpretation of *La Géométrie* just stated, the only reason Descartes needs to exclude curves that he claims are mechanical is that he doubts their certainty. Thus, he does not need proof to exclude them. This also explains why Descartes' classification was ul-

timately incorrect. While he excluded curves that he considered to be doubtful, some were later determined to be exact. In spite of this, the fact that Descartes excluded these curves from his geometrical studies is ultimately irrelevant because he would have been specifically focused on curves that he himself knew were exact and therefore geometrical. In mathematics today, we use a modified version of Descartes' classification, for which we have Leibniz to thank. The current classification is one between algebraic and transcendental curves. As defined by Leibniz, algebraic curves are those represented by an equation of certain order (comprised of a certain number of parts) and transcendental curves are those represented by an equation of indefinite or infinite order.

We can also understand Descartes' attempt at creating an upper bound for the number of "real" solutions to an equation based on the rule of signs, if we first consider the method of the *Discourse* and then question how Descartes used this method as it relates to negative and complex numbers. As mentioned earlier, the only reason Descartes needed to reject negative and complex solutions to an equation is the fact that the notions of negative and complex numbers are vague and therefore, according to Descartes, not certain. As with his classification of geometric and mechanical curves, the false results obtained by Descartes have been elevated in importance to a position much higher than the problem of determining how he used his method of systematic doubt to obtain these results. We noted that Leibniz corrected and finished Descartes' work on curve classification. It was Newton who corrected Descartes' flaws in finding how many positive integer solutions exist for specific algebraic equations. Newton included the correction in his *Arithmetica universalis*. Today, Newton

is best known for his *Philosophiae naturalis principia mathematica*, his magnum opus in which the differential calculus was formally outlined. However, in Newton's day, *The Arithmetica*, along with its correction of Descartes' rule of signs, was printed more than any other of Newton's works.<sup>13</sup>

One of the many oddities that many mathematicians today find in Descartes' writings is his rejection of mathematical proof by contradiction. (Some mathematicians, namely the constructivists, take a similar stance today.) Descartes stated that such a proof is "without intelligence and simply by chance." Again, it is in the *Discourse* where we find the reason why: he doubted the method of proof by contradiction (this notion, while modified, still plays an important role in mathematical logic, especially in relation to claims about infinity), and therefore refused to use it as the basis of certainty. While modern practices in mathematics rely on proof by contradiction quite heavily, we should emphasize the fact that Descartes rejection of this method had a very profound impact on advances in geometry and algebra. Not only was Descartes very systematic in his formulations of algebraic geometry, he concentrated specifically on gathering as much information as possible from the problem at hand before even attempting a solution, then used this information to either find a solution or determine that none was achievable. While many mathematicians failed to realize the subtleties of the method he was using, the attempts to rectify Descartes results lead down a path that was very similar. In other words, in failures to understand the theoretical aspects of Descartes' program, mathematics was pushed towards a method of systematic doubt, a method very similar to that of the *Discourse*, which would become the basis of analytic geometry.

The question can therefore be stated: "Is Descartes the father of modern analytic geometry?" While the answer to this question is more complex, we can assert a general reply in the form of, "Yes, but more by accident than by intent." My main argument here is that instead of studying the reasons why Descartes wrote *La Géométrie*, related it to the *Discourse*, and what originally motivated him to argue against the skeptics with a treatise

on geometry and algebra designed for the general educated public, the majority of efforts to interpret the work seem to be focused more on the mathematical, geometric, and algebraic results it contains. After all, we seem to have a tendency, at least in the discipline of mathematics, to view both progress and history in mathematics as a series of results and discoveries. As mentioned earlier, philosophers of history such as Adorno and Foucault warn us about such assumptions of continuity in history. It would appear that more concentration needs to be placed on the discursive practices of the time that actually lead, or at least influenced, mathematicians to formulate their theories. Mathematicians don't currently concentrate on this at all; they never really have.

*La Géométrie* was translated to Latin and published in four editions: 1649, 1659-61, 1683, and 1693. A commentary by Van Schooten was added to the second Latin edition in order to clarify as much as possible. (It was this version that Newton is said to have read.)<sup>14</sup> The remarks themselves were over twice as long as the original edition of *La Géométrie* and were written as introductions to Descartes' horribly organized results and methods for mathematics students. In each of the later editions other comments were added. To this day, the work is rarely explained in any fundamental way as being part of the *Discourse* and attempts to understand the mathematical content have led to varying interpretations of what Descartes' goals in writing the work actually were.

In order to understand *La Géométrie*, it is important that mathematicians consider that it was originally intended as an example of methodology. Therefore, in order to appreciate this work, I argue that mathematicians should concentrate on an interpretation of the *Discourse* before venturing on to *La Géométrie*. Before appreciating the *Discourse*, the reasons for its publication (along with Descartes' relation to the skeptics and religious institutions) must be noted without falling into common assumptions (i.e., doubtable preconceptions). Similarly, those interested in Descartes' philosophy should consider the broad range of applicability which Descartes himself presented for his method with the three appendages to the *Discourse*. In fact, the *Discourse* itself can be seen as an introduction to three examples on how certainty can be found in scientific reasoning.

While *La Géométrie* has been incorrectly interpreted for the most part, the profound influence that Descartes has exercised upon the mathematical world can be seen as a direct consequence of these interpretations. This is perhaps the greatest of ironies surrounding modern Cartesian scholarship.

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## NOTES

<sup>1</sup> Burton, 329.

<sup>2</sup> Anglin, 158.

<sup>3</sup> Newman, 236.

<sup>4</sup> Foucault, 21.

<sup>5</sup> Adorno, 139.

<sup>6</sup> Boyer, 335.

<sup>7</sup> Foucault, 25.

<sup>8</sup> Burton, 330.

<sup>9</sup> This problem states: Given three or more straight lines, to find the geometrical locus (the set or configuration of all points whose coordinates satisfy a single equation or one or more algebraic conditions) such that, if one draws from the points rectilinear (straight) segments cutting the given straight lines at given angles, the product of two of these segments will be equal to the third or to the product of the third by the fourth, and so on. A version of this problem (in modern mathematical notation) is to find the roots of the equation  $y = (x^3 - 2x^2 - x + 2) / x$ .

<sup>10</sup> Boyer, 344.

<sup>11</sup> Burton, 337.

<sup>12</sup> Mancosu, 77.

<sup>13</sup> Boyer, 412.

<sup>14</sup> Boyer, 391.