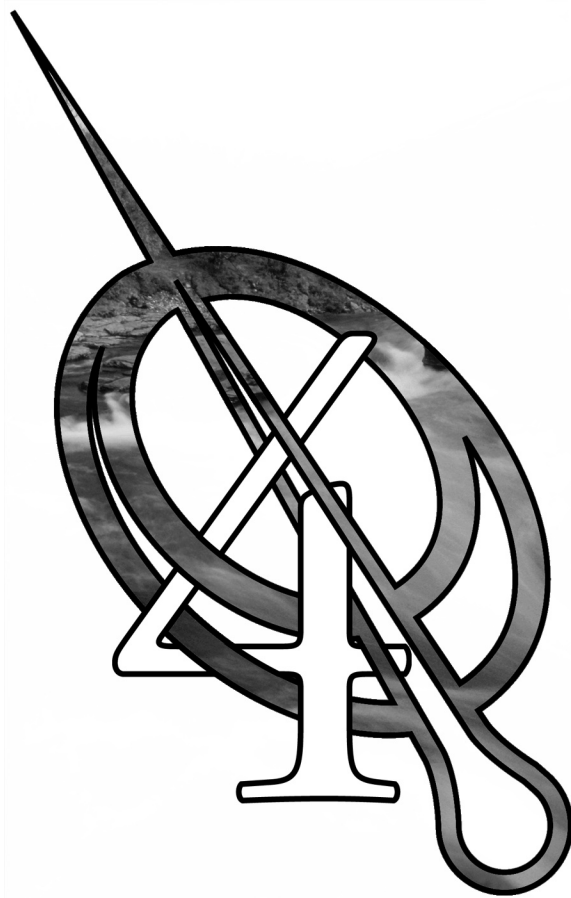


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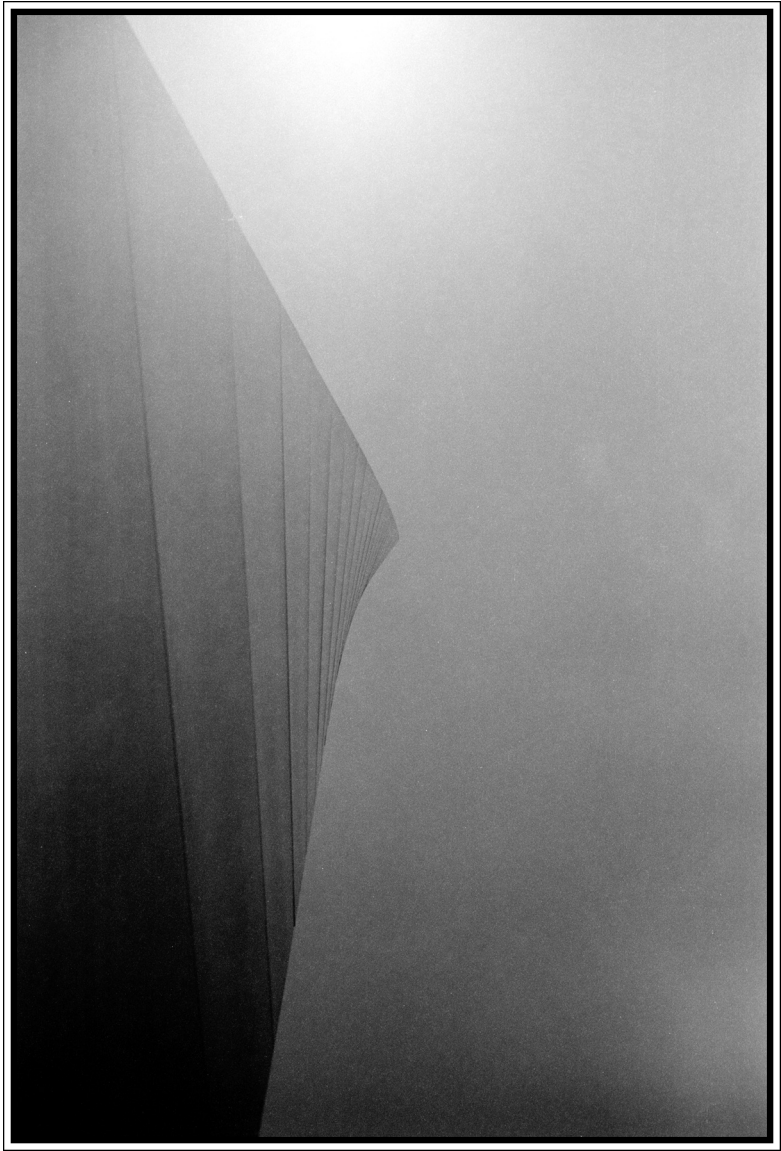


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Vagueness: Language vs. Logic

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- § 1 Timothy Williamson notes that vagueness has been perceived as a problem by the two philosophical traditions highly concerned with formal logic, the Stoics and modern analytic philosophy.¹ This is not surprising, for the problem of vagueness that language presents can be seen most readily in the sorites paradox. This paradox presents a series of predications that seemingly command assent, e.g. A man with one hair on his head is bald; If so, then a man with two hairs on his head is bald; If so, then a man with three hairs on his head is bald; etc. Since the starting point is true, and since we can never confidently say which hair added makes him no longer bald, we are left falsely concluding that a man with thousands of hairs is bald. While any language user off the street could be confronted with this seeming incoherency generated by vague predicates, the significance of most everyday discourse is preserved unblemished. Though I couldn't identify the exact point on a color spectrum at which red ends and yellow begins, this worries me not when I see a fire truck drive by. Only those, that is, who are concerned to correlate formal logic and natural language, would find the sorites paradox's implications disquieting and globally important. The exact nature of vagueness and the problems to which it gives rise are the subject of section II.

There are two main ways to avoid the logical contradiction created by vagueness: revise either classical logic or the functioning of our vague predicates. There have been three attempts at the first approach: supervaluationism, three-valued logic, and fuzzy or degree logics. The second approach, called the epistemic view, is taken up by Williamson, who proposes instead that we do not think of our concepts as vague but rather our epistemic state as ignorant. There is, however, a final approach, that taken by Bertrand Russell, who rejects the applicability of an exact logic to our inherently vague language. In a sense, Russell is siding with the vast majority of humanity not obsessed with relating formal logic and natural language; he creates a rift between the two. Two criteria guide the search for an answer among these five options: (1) is the makeup of our conceptual structure, and (2) is the functioning of that structure in relation to the world, i.e. the cognitive status of propositions, or the truth of vague statements. In section III I use criterion (1) destructively, rejecting solutions one through four and agreeing with Russell that vagueness, inherent in all our language, prevents it from ever squaring with formal logic. But in section IV I perform a constructive task with criterion (2) by incorporating a fundamentally important insight of Williamson (stemming from Tarski's theorem) that qualifies how we relate truth to vague statements; inasmuch as all statements of our natural language are vague, this means drawing out general implications for a theory of truth.

§ 2 There are two conditions for vagueness: a term is vague if and only if (i) it has borderline cases, and (ii) it has higher-order borderline cases.² "Borderline" means that, in one or more instances, one is unable to decide whether or not a predicate applies to a particular object. For example, the predicate "house" is vague because there are certain structures, e.g. four walls without a roof or a roof with three walls, concerning which I wouldn't be able to say whether or not they are houses. Condition (ii), higher-order vagueness, means that it is also unclear as to where the unclarity over the applicability of a predicate starts. One could call it the existence of borderline borderline cases. For example, on a color spectrum between red and yellow there is an interim zone, call it "orange", for which one cannot decide whether a particular shade is red or yellow. But one can further not tell where the zone of in-between itself starts, e.g. a case between red and orange. These two conditions are necessary and sufficient for the sorites paradox, because they entail the indistinguishability of the extension of a predicate along a continuous spectrum.

In elucidating “borderline”, we must be careful not to exclude definitionally the epistemic view, which question-begging would occur if we said that vagueness occurs iff our ignorance does not prevent us from knowing whether or not the predicate applies.³ The nuance between these two types of definition is highlighted by properly distinguishing between vagueness and underspecificity. Underspecificity occurs when, by simply knowing more about the exact state of the physical world, we would then be able to unambiguously apply a predicate. Assume, for example, that 500 or more people constitute a mob. Glancing at a crowd of 501, we might not be able to decide whether or not to call it a mob. This undecidability is underspecificity and not vagueness, however, because by counting, i.e. refining the exactness of our knowledge about the world, we could then decide whether or not to call the group a mob. Hence this predication is true, even though at a glance we might not be able to say so.

Vagueness results, in contrast, when our margin of error in specification is greater than the level knowledge possible. Continuing the example, to gauge the underspecificity of “mob”, we assumed each person to be an atomic unit. If borderline people are allowed, however, we could not conclude, as above, that we would be able to decide the applicability of the predication simply by knowing which atoms of the universe are where. Vagueness, then, is a conceptual or representational matter when no more exact knowledge of the world would resolve our inability to decide upon the predicate’s applicability. It is an additional and important question as to whether there is still a real borderline to our vague concepts that we cannot know because of this margin of error (as Williamson contends). We could summarize that vagueness is the inability to apply a predicate (to borderline cases) not based on underspecificity. Notice how “underspecificity” has been substituted for “ignorance” to rectify the question-begging definition.

One question worth pursuing is, Where does the root of vagueness lie? The first option is tracing it to the world itself. Differentiating underspecificity from vagueness, however, already signals the dead-end of this path. At first sight we might think that vagueness possibly occurs in the world on two levels, the macro or the micro. In truth, however, both are forms of underspecificity. On the macro level, many phenomena appear to lack definite boundaries, e.g. a cloud. If we use a deeper level of exactitude, however, such as the atom or molecule, we see that this phenomenon is merely underspecificity. With every air molecule differentiated, it is then a mere conceptual matter as to where one draws the boundary of the cloud. Similarly, it is a misun-

derstanding to call the phenomenon of a mountain vague: the matter constituting each ridgeline and gully is itself clearly identifiable; it is only a conceptual vagueness that wavers as to where to demarcate the mountain's boundary.⁴

The micro level may appear more complex, but the quantitative situation is no different. Firstly, the apparent infinite divisibility of matter points to the issue of underspecificity, but not necessarily to vagueness. For example, light forms a continuous spectrum that lacks any internal quantitative boundaries, but that in no way affects the fact that a certain wave has a certain measurable wavelength. Secondly, on the micro level some phenomena cannot possibly be exactly measured, as demonstrated by Heisenberg's Uncertainty Principle, but that very principle entails positing a position and velocity for a particle that we cannot observe without altering. If vagueness does not reside in the world, it must then somehow be connected to our representation of the world.⁵

The sorites paradox uses this conceptual fuzziness as its launch pad. In antiquity the paradox was formed as a series of questions. The archetypal case is the heap, from which the name "sorites" comes ("soros" being Greek for "heap").⁶ The accosting philosopher would begin by asking whether one grain of wheat makes a heap. The answer: Certainly not. But then, he continues, do two grains make a heap? Do three? Four? Five? The questions can be extended ad infinitum and, if we are never willing to admit that the marginal addition of one grain makes the difference, we are left concluding that, no matter how large, no pile of wheat is a heap.

A structure similar to the Q&A can be generated in the form of a logical argument:

- (1) 1 grain isn't a heap.
- (2) If 1 grain isn't a heap, then 2 grains aren't a heap.
- (3) If 2 grains aren't a heap, then 3 grains aren't a heap.
- ...
- (10⁶) If 999,999 grains aren't a heap, then one million grains aren't a heap.

Therefore, one millions grains aren't a heap.⁷

The importance of this form of the argument is its simplicity. With just modus ponens we apparently derive an absurd falsity from true premises. While simplicity makes the argument's validity difficult to challenge, the truth of the premises is equally defensible. Premise one looks uncontested. Premises two through one million are at least prima facie

open to question, because we can recognize the quantitative addition of another grain. But the difficulty of making a judgment of predication based on a marginal distinction can easily be magnified. Take a sorites series of height, for example: from a one-meter-tall person who is clearly short, we could prove that someone ten meters tall, or even one million meters tall, is also short. Here, however, we can decrease the marginal change between steps while simultaneously increasing the number of premises. Instead of the liberal difference, “If someone 1 meter tall is short, then someone 1.1 meters tall is short”, we could state “... then someone 1.01 meters tall is short”, or “... then someone 1.000001 meters tall is short.” It’s no longer clear that one can contest such premises, because at the limit one could make the marginal difference immeasurable: with infinite premises we could reduce the marginal change to infinitely small. If one allows the coherency of arguments with infinitely many premises, this would make it nearly absurd to contest any of them based on an infinitely small change.

Besides this first formalized argument, there are two additional important logical forms of the paradox. The second condenses the conditional statements into a quantified premise. For the predicate F (short, for example) and the sequence of objects x_i , the paradox asserts:

- (1) Fx_1
- (2) For all i , if Fx_i then Fx_{i+1}
and therefore, (3) Fx_n ⁸

Premise (1) is an empirical observation, as is conclusion (3), which contradicts the fact that x_n is actually ten-meters tall, for example.

The third form simplifies the premises and generates a quantified conclusion:

- (1) Fx_1
- (2) $\sim Fx_{1,000,000}$ therefore
- (3) There is some x such that Fx and $\sim Fx_{i+1}$ ⁹

Given bivalence, this argument is valid. The falsity of this conclusion is, according to many commentators, entailed by the nature of vagueness: because predicates like “bald” are vague, there is no single hair (x_{i+1}) that makes someone no longer bald. (Note that the conclusion’s truth is precisely what the epistemic view argues, so this form of the argument is a paradox for everyone else but is a “sound” proof for the epistemic view’s proponents such as Williamson.)

We might wonder, however, whether the sorites paradox is actually a serious problem. Pragmatically, it does not hinder language usage, nor

does it reduce or erase the putative meaning of vague terms. There are at least three reasons to worry. One reason is that we cannot accept the conclusion, that it's just "strange but true" that one million grains don't make a heap.¹⁰ This complaisance will not do. Because the paradoxes are reversible, we could start with the observation that one million grains make a heap and with equal soundness prove that one grain of wheat also makes a heap. We could show that red is yellow or that yellow is red. Without direction or a way out of the vicious derivations, complete logical meaninglessness would result.

A second cause for concern is the pervasiveness of vagueness in our language. Nearly all parties agree that when natural languages are scrutinized, vagueness is endemic and perhaps universal. There are many specific concepts, however, for which we desire a more accurate account and exact borders, e.g. moral and legal concepts such as personhood and responsibility. In order to perform tasks with these words, we need some way of creating borders and resolving vagueness (i.e. a way that is helpful and/or true).

The final reason, persuasively argued by Williamson, is the importance of classical logic to analytic philosophy and its understanding of truth, namely Tarski's theory (discussed below). Retaining bivalence (or some essential semblance *apropos* of Tarski) and avoiding the sorites paradox could be a litmus test, then, for an explanation of vagueness, what I called above criterion (2). Next we turn to the responses to our problem.

§ 3

Of the five primary answers to vagueness, the first is supervaluationism, which identifies vagueness as a deficiency in a predicate's meaning. In response, this approach uses a bivalent, non-vague meta-language to give a more precise account of the meaning of vague predications. Supervaluationism, explored extensively by Kit Fine, works by examining all the possible "precisifications", or specifications, of a predicate, which are the ways of drawing a sharp boundary for it. For example, the predicate "bald" could hypothetically be made precise if we decided that a person with 100 or less hairs is bald, but 101 hairs or more makes him or her non-bald. All predications would now have an unambiguous truth-value. The choice of 100, however, is arbitrary, since the conditions of vagueness entail not knowing such a cutoff point. Supervaluationism demands that we take into account all such possible ways of precisification, and forms a new super-truth-value (which I will here call the truth-value*) based on the results. A sentence is defined as true* if and only if it is true on all such precisifications, false* if false on all, and otherwise neither* (true nor false).¹¹

This solution provides a simple and intuitive explanation for the first two forms of the sorites paradox. In the first form, for any precisification, the conditional at that point will fail. In our example, “If a man with 100 hairs is bald, then a man with 101 hairs is bald” comes out false, because the antecedent is true but the consequent false. Since this conditional will be true on other precisifications, e.g. where 102 hairs starts the point of non-baldness, it gets a super-value of neither*. For the same reason of conditional failure the second premise of the second formal argument is also rejected, occluding the reductio. These results accord well with our understanding of vagueness as entailing borderline cases for which we’re not sure that predications should have a definitive value of true or false. An additional benefit is that super-valuationism preserves much of classical logic, including the principle of the excluded middle, since “ p or $\sim p$ ” will come out true on all precisifications.¹²

Supervaluationism has three critical problems, however, that prevent it from adequately explaining vagueness. Firstly, this theory does not preserve bivalence in the object language, since a borderline case of tall will come out true on some and false on other precisifications. The importance of this failure for a theory of truth will be discussed below in section IV. Secondly, higher-order vagueness is left unaccounted for. Any sentence of the object language will be given a definite super-value, so there will be a sharp cutoff between the true* cases and the borderline cases of neither*. Concerning the reality of vagueness: unfortunately it ain’t so. Lastly, supervaluationism’s non-classical semantics fails to reflect our conceptual structure. This can be seen in its failure to explain the falsity of the conclusion in the third form of the paradox. It holds, for example, to the truth of the statement “There’s a hair that makes all the difference to baldness”, but the falsity of such a sentence seems precisely what an adequate account of vagueness is supposed to explain.¹³

The second answer to the paradoxes is three-valued logic (or any finite-valued logic greater than two), which posits a value of indefiniteness in addition to truth and falsity. The logical connectives are then redefined for this non-bivalent metalanguage. Negation, for example, is true if and only if the component is false, but, if the component is indefinite, its negation is indefinite as well.¹⁴ Three-valued logics answer vagueness by making logical predication likewise sensitive to the indefiniteness of some semantic facts: since we are committed to vagueness in our object language, wouldn’t a metalanguage that incorporated indefiniteness provide the best model? This approach does

escape the sorites paradoxes, for the conditional premises come out indefinite rather than true; thus validity goes by the wayside.

The answer to the above question, however, is that a three-valued logic does not provide the best model for our vague language. First, and just as in supervaluationism, the failure of bivalence on account of three semantic values counts strongly against this view, as discussed below. Second, this approach commits a fundamental error in its modeling strategy: although the metalanguage is non-bivalent, it is not vague. The theory, that is, gives definite semantic values to all predications of the object language, even if it admits that one of these values is itself indefinite. We can see, then, why this approach has a difficulty with higher-ordered vagueness. There will be sharp lines between true statements and the statements valued indefinite as long as the semantic theory rigidly attaches values to the predications. It therefore cannot explain vagueness.

Fuzzy logic or degree theories are the third answer. Degree theories take a very different approach to explaining vagueness than multivalent theories: the former don't try to mirror the object language's seeming non-bivalence, but instead play off the comparative nature of the object language. Two women might both be borderline bald, but we could nonetheless say that one is balder than the other. Simple semantic indeterminacy, these theorists say, isn't the point at issue. To the question, "Are these two women bald?", we can imagine saying, "Sort of, but one is balder than the other." The degree theorist claims that this means we could say then that "The one is bald" is more true than "The other is bald." This is a controversial step, and one for which I don't have much sympathy (though degree logics may be extremely worthwhile for other purposes). The specifics of this approach are as follows. Degrees of truth can be quantified by probabilities between 0 and 1, where these boundaries mark falsity and truth, respectively. The logical connectives are then redefined, e.g. the truth of a negation becomes one minus the probability of its component: $\sim p = 1 - p$.¹⁵ A falsity (0) negated becomes a truth (1), and the middle of the road 0.5 negated is also 0.5. There are two different ways to define validity for such a system. By truth-preservation models, for a valid inference the conclusion must have a truth degree no lower than the lowest premise. Alternately, validity could be conceived of as allowing no more falsity in the conclusion than the sum of the premises' degrees of falsity. Either of these conceptions provides explanations of the paradoxes: for the first two forms of the sorites, the conditional premises will have consequents that are progressively less true (or more false) than the anteced-

ents, so soundness is lost; the third formalization is also clearly invalid, for the two premises have a truth degree of 1, while the quantified conclusion's value is only about 0.5.¹⁶

These results are encouraging, but problems are rife for degree theories. Firstly, higher-order vagueness is just as inadequately explained here as by the previous two theories, because of the absolute break between the truth of probability 1 and the near truth of 0.999. Second-order vagueness entails the impossibility of this distinction, for we cannot identify a sharp boundary between clear and borderline cases. Secondly, degree theories yield some extremely unintuitive results, raising doubts about their explanatory adequacy. For example, when the probability of "p" is 0.5, the statement "p or ~p" is as true as "p & ~p". Lastly, it's not entirely clear that degree theories fully explain the third logical form of the paradox, for the falsity of its conclusion (that predicates have sharp boundaries) is seemingly entailed by vagueness, but these theories give it a value of 0.5.¹⁷

The fourth answer is Williamson's epistemic view, which holds vagueness to be a form of ignorance about our concepts, for, it contends, those concepts themselves actually have sharp boundaries and well-defined extensions.¹⁸ This answer preserves classical logic, and escapes the first two forms of the paradox: in form one by denying the truth of one of the conditionals, and by denying premise two in the second form, since there is a sharp boundary that prevents the reductio even though we do not and perhaps cannot know where it is. The third form, additionally, the epistemic view takes as a sound proof, since the concluded existence of a sharp boundary is precisely what it argues for. Williamson can similarly use the first two logical forms to his advantage: because they are valid, and because the first premise and conclusion are both true, we have a reductio ad absurdum for the failure of one of the conditional premises.

The epistemic view was also the Stoic's reaction to the paradox. Because they accepted both logic's applicability to language and the principle of bivalence, the Stoics reasoned from the reductio to the falsity of one of the conditionals, and concluded that predicates have sharp boundaries that we are ignorant of. According to the Stoic ideal of wisdom, one would only speak truth: confronted with the paradox, then, the Stoic Chrysippus recommended falling silent, in the face of one's ignorance, well before one is in danger of crossing the sharp boundary and asserting a falsehood.¹⁹

The epistemic view retains both classical logic and semantics, and is therefore not subject to any of the criticism of the previous solutions.

It has, however, several hurdles of its own. For one, it must explain the counterintuitive notion of ignorance that is essential to its success. Williamson tackles this challenge in two ways. He contends, firstly, that ignorance should actually be seen as our default state, defending our lack of knowledge from the need of explanation.²⁰ Secondly, he explains our ignorance with the notion of a margin of error mentioned in section II. Human cognitive faculties are not reliable enough to make distinctions commensurate with the infinite underspecificity of the natural world. We simply cannot distinguish, for example, between extremely fine shades of red.

This explanation of our ignorance both helps and hurts Williamson's cause. It is certainly true that our finite, perspectival cognition is not up for complete or precise comprehension of the natural world: absolute or objective knowledge is a chimera, and therefore ignorance is very much a constitutive factor of human life. This ignorance, however, also limits our discourse, and here Williamson runs into difficulty. Here I follow the general philosophical tradition of Wittgenstein, who argued that there cannot be a difference in meaning without a difference in use, and who similarly rejected "the notion of imperceptible rails upon which the correct usage of language is supposed to run."²¹

Here is the problem. Williamson fully agrees that meaning supervenes upon use, so that "there is no difference in meaning without a difference in use."²² Our actual use of predicates, however, treats them as if they lack sharp boundaries. This is precisely why the paradoxes arise. If a predicate has only as precise a meaning as usage has conferred upon it, or, to put it another way, if it only has a content garnered from its instances of usage, then it seems flatly wrong to insist that it has an exact extension.

An example will make this clear. Say I create the new word "strumpity", and use it once while looking at a dog walk across the lawn, in the sentence "That dog is strumpity." Williamson must maintain that by fiat this word has sharp boundaries.²³ Even if we add bits of intensional discourse, such as that I intend by "strumpity" some sense similar to but not exactly the same as "short", my neologism would still have no sharply extended meaning content, which according to Williamson it must have. Where would the predicate's boundaries come from? Williamson does not answer this question, but asserts that it is still possible that such boundaries exist. In one place he remarks that nature could contain such boundaries: e.g. since a heap cannot be made from a flat layer of grains, at least four grains are required to make a heap, "three grains close together supporting a fourth on top."²⁴

His argument here, however, is misguided. The point is not whether the world contains any boundaries reflected in our concepts. There certainly are boundary-marking features of concepts on account of the world, for example the boundary between molecules of O_2 that do or do not have a hydrogen atom attached. But asserting this feature of language does not discount the simultaneous existence of conceptual vagueness in language. We can concede to Williamson that usage might reflect some sharp boundaries in the world, but the point remains that our concepts' boundaries are not completely sharp. Vagueness resides in our representation of the world, which means we have to look to how that representation itself operates. And this returns us to the supervenience of meaning upon use. If the extension has not been gotten by stipulation or use, a full account of meaning (to Williamson's purposes) with the supervenience principle is occluded: Williamson can only manufacture boundaries out of thin air, and this is unacceptable.

Besides the logical arguments for the epistemic view, much of Williamson's clout comes from the untenability of an adequate alternative explanation of vagueness (which would be overcome if the analysis given below is tenable), but we should not confuse this with an adequate description of the functioning of our predicates. On account of the trouble with meaning and use, I consider the epistemic view as unsuccessful as the previous three solutions. Now to the alternative.

§ 4

To reject all of these explanations and conclude that natural language simply does not conform to formal logic is to side with Russell, answer number five. If one takes this view, one can still retain logic for its usefulness in specific cases, mining various systems pragmatically as one uses mathematics in physics. One such appropriation is classical logic and bivalence in connection with truth: by establishing some connection between the cognitive functioning of language and logically formalized predication, we can salvage cognitive meaning and avoid what Williamson calls nihilism, the condition resigned to the meaninglessness of vague statements.²⁵

The question is whether we can afford to agree with Russell and create a rift between language and logic. Williamson's primary concern is to prevent this separation, and he makes much of the point that for pragmatic reasons alone we could justifiably side with his view as the best explanation of vagueness. Why? Because, he correctly observes, analytic philosophy is committed to an understanding of truth that incorporates or at least resembles Tarski's theory, and, as Williamson (and Tarski) has shown, this entails bivalence or something like it.²⁶ A

complete rift between logic and language, then, would derail the truth-functioning of language.

The importance of Tarski's theorem here is its connection to bivalence, viz. Tarski's theorem coupled with a rejection of bivalence creates a logical contradiction. Tarski's theorem defines truth predication by disquotation, e.g. "The sentence 'it is raining' is true iff it is raining." Formally expressed, where T is the truth predicate and p = it is raining, this sentence becomes:

$$(1) T"p" \text{ iff } p.$$

A second example is:

$$(2) T"\sim p" \text{ iff } \sim p.$$

The denial of bivalence can be expressed with the left-hand quantities of these biconditionals, asserting, e.g., that "It is not the case that ('It is raining' is true or 'It is not raining' is true)":

$$(3) \sim(T"p" \text{ or } T"\sim p").$$

By substitution of each of the quantities in this disjunction (3) with their right-hand partners from the biconditionals (1) and (2) above, we then get:

$$(4) \sim(p \text{ or } \sim p).$$

By De Morgan's law's, (4) can be transformed into:

$$(5) \sim p \ \& \ \sim \sim p.$$

Here we have the contradiction, whether or not double negation can be eliminated.²⁷ I consider this proof as important as Williamson does: whether or not truth simply is the disquotational process of Tarski's theorem, I think it would be difficult to say what truth is without at least admitting that "It is raining" is true if and only if it actually is raining. As Williamson observes, this logical contradiction is unacceptable. It counts against Williamson's opposition by catching the Russellian view in a sort of liar's paradox, in which Russell would be saying something true, viz. that language does not align with logic, that would thereby render truth-saying impossible.

Assuming the analysis of criterion (1) above is correct (section III), that the epistemic view does not really do justice to the vague status of our conceptual structure, should we nonetheless concede with Williamson that we must posit some unknown and perhaps unknowable epistemic status to vague concepts? No, so long as we can bridge the gap between logic and language by explaining the cognitive functioning of language, criterion (2).

A brief sketch of such an explanation follows, based on the distinction noted in section I, namely that between (1) the psychological structure and interworkings of our concepts, and (2) those concepts' external functioning vis-à-vis the world, i.e. their cognitive functioning or truth. Williamson's fundamental insight for (2), which he mistakenly assimilates with (1), is that any predicate used cognitively, i.e. used to say something about how the world is, can only be applied bivalently to an object.²⁸ For example, take the predicate Fx , where F means "x is a mug". Where $a = \text{me}$, Fa is false. Now for $b = \text{this coffee mug on my desk}$, Fb is true. So far so good. Next imagine a third case where $c = \text{my Chinese teacup that lacks a handle}$, i.e. a borderline case that may or may not be a mug. My contention—contra Williamson—is this: to maintain Russell's position, we must only hold that by saying Fc is neither true nor false one is no longer saying anything about the world. This maintains the necessary condition that vague predicates, used cognitively, must admit of analysis that conforms to Tarski's theory and therefore bivalency.²⁹

Is this copout acceptable? Yes, because Williamson grants, correctly, that intuitionism in mathematics is perfectly reasonable. He states: "the argument [reducing denials of bivalence to absurdity] applies not whenever bivalence is denied, but only when it is denied of a particular sentence."³⁰ But I am perfectly happy to grant that some sentences are used non-cognitively and have nothing to do with bivalency: the undecidability of applying a predicate seems to be exactly this. Completing this analysis with regard to a complete theory of truth would require further investigation than this paper allows, but we can note: one, saying "definitely short" seems to only differ with "short" through noting the subject's commitment (e.g. epistemic) to the application of the predicate, not a cognitive difference in the world or degree of truth within the predicate itself; and, two, saying "somewhat short" or "vaguely short", where one intends a predication, must only be seen as a slightly different predicate (or function) within the amalgam of our conceptual system—one can use it cognitively, but thereby one uses it differently or uses only a related predicate.³¹

The explanatory power of this nuance, in addition, makes the amended Russellian view superior to any other theory of vagueness. Take, for example, the case of three people in a row, #1 who is clearly short, #2 who is a borderline case, and #3 who is clearly not short. Contra the non-classical logical replies that assign a third status to the borderline case (whether it be absolute or of a degree), by withholding judgment for person #2 we are precisely declining to make a cognitive assertion about him. Just what tertium quid status we would be assigning is difficult to imagine with the importance and basicity of Tarski's theory in view.

With this explanatory device for the Russellian answer to vagueness, can we give a more detailed account of the sorites paradox? Yes. In the second scheme, for the predicate F and the sequence of objects x_i , the paradox asserts:

- (1) Fx_1
- (2) For all i , if Fx_i then Fx_{i+1}
and concludes
- (3) $Fx_{1,000,000}$

Premise (1) and conclusion (3) are cognitive assertions, true and false respectively, to which bivalency applies. Premise (2), however, is an assumption about the psychological structure and status of our concepts which we can join Russell in rejecting. Since all of our language and concepts are actually vague, this logical predication of their relation is false and commits a category mistake. And we now have a similar explanation for the falsity of the conclusion in the third scheme: that there is a hair that makes all the difference to baldness is flatly contradicted by our nebular concepts.

§ 5 Does vagueness in natural language give rise to a philosophical problem? Yes. Is it soluble? Yes. The sorites paradox gives us good reason to doubt that any formal logic can precisely model natural language. We can view various logical systems as attempts to capture a particular aspect of language as adequately as possible. Although none is perfect, we can appropriate them based on their usefulness and success. In the case of vagueness, none properly explains the issues at hand, because logical predication requires sharp boundaries, precisely what concepts of natural language lack. We can nonetheless, pace Williamson, use classical logic to the extent that it helps us understand the truth-functioning of our cognitive utterances. This relation between language and classical logic's bivalence principle is further brought to light by an examination of the vagueness of predicates and our conceptual structure, particularly in contrast to the underspecificity of the natural world.

Concerning this last point: our world often has exact boundaries given the precision of our unaided senses, e.g. I can clearly see the extension of this coffee mug, where it ends and my desktop begins. If we thought that the objects of the world correspond via words to ideas in our conceptual apparatus à la empiricists Locke and Hume, then it would be natural to posit an at-least theoretical analogue to under-specificity in our minds. Williamson, in his approach to the problem of vagueness, makes a similar shift from the outer world to the inner mind (not to attribute a philosophy of Ideas to him, though). However, if the reification of concepts is broken down to use, e.g. following something like Wittgenstein's concept of family-resemblance, we would no more expect all predications to be logically precise than we would expect all arguments to be valid or all truths derived from clear and distinct ideas.³²

Notes

- ¹ Williamson (2001) 36.
- ² Keefe and Smith, 2; Additionally, One might think that there is a more moderate form of non-sorites vagueness constituted only by condition (i); as such it would simply yield a non-binary system, and would not be the type of vagueness inherent in natural language, as explained below.
- ³ Williamson (1999) 270.
- ⁴ Cf. this error in Tye and Sainsbury (Keefe and Smith 17).
- ⁵ I am in complete agreement on this point with Russell. See Russell 62-3 and 68, and Williamson (2001) 52-3.
- ⁶ Sainsbury and Williamson, 458.
- ⁷ Sainsbury and Williamson, 462.
- ⁸ Keefe and Smith, 9-10.
- ⁹ Sainsbury and Williamson, 468.
- ¹⁰ Sainsbury and Williamson, 465.
- ¹¹ Keefe and Smith, 23.
- ¹² Sainsbury and Williamson, 472.
- ¹³ Keefe and Smith, 32; One way to extend the case for supervaluationism is to incorporate a definitely operator, "Def", as Fine himself does (Fine 140; Sainsbury and Williamson 473-4). Since the resultant complications are outside the scope of this essay, I will only note that such a move cannot account for higher-order vagueness and therefore need not worry us for the present purposes.
- ¹⁴ Sainsbury and Williamson, 478.
- ¹⁵ Sainsbury and Williamson, 476.
- ¹⁶ Sainsbury and Williamson, 476-7.
- ¹⁷ Sainsbury and Williamson, 476-7; One could amend this problem with a definitely operator, but in light of the other difficulties that void the solution-check written by degree theories, it's not necessary to pursue the vicissitudes of this objection.
- ¹⁸ Keefe and Smith, 17.
- ¹⁹ Sainsbury and Williamson, 459-61.
- ²⁰ Keefe and Smith 19.
- ²¹ Sainsbury and Williamson 479; Wittgenstein sections 65-6, et al.
- ²² Keefe and Smith, 19.
- ²³ Keefe and Smith, 21.
- ²⁴ Sainsbury and Williamson, 480.
- ²⁵ Williamson (2001) 165ff.
- ²⁶ Williamson (1999) 268.
- ²⁷ Proof reproduced from Williamson (1999) 265-6.
- ²⁸ Williamson (1999) 265-6.
- ²⁹ An interesting connection would be between the seemingly mild semantic indeterminacy entailed by my view here and the work of Quine. Perhaps there is a worm of indeterminacy at the core of the semantic apple. But that will have to await another essay.
- ³⁰ Williamson (1999) 266.
- ³¹ E.g. Maybe "somewhat short" should be treated as essentially comparative (to some norm, for example), and closer in form to a two-place predicate (such as Sxy meaning "x is shorter than y") rather than as an indexed truth-degree of the one-place predicate "short".
- ³² I would like to acknowledge the help of Elizabeth Fricker, the tutor for whom I originally handed this essay in, and the help of the other professors and students who gave me the benefit of their comments: Bill Mander, Anamitra Deb and Benjamin Rusch.

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